# Optimizing Model of Decision Making

Tomasz Smoleń, Jagiellonian University, Kraków

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## Summary

The unifying mathematical model of decision making is proposed. The model is based on maximization of an expected gain, achieved by optimizing the amount of information perceived from the environment, in which decision is taken. The mechanism of the model assumes the existence of a trade-off between maximization of a reward value and maximization of the probability of choosing the proper alternative. The former goal requires minimization of the amount of information gained from the environment, whereas the latter one requires maximization of the amount of gained information.

## Theoretical Base

An attempt to verify the Optimizing Model of decision making was, in fact, underlain by an intent to verify two more general hypotheses: (a) that human behaviour and cognition is rational (optimal) in the Anderson's (1991) sense, and (b) that there exists one, general and flexible strategy of choosing the best alternative on the basis of multiple cues.

Newell (2005) has explicated a thesis that all strategies from the adaptive toolbox can be represented as one strategy, with continuous parameter, which determines, how much information is gained from the environment. This interesting proposition is purely theoretical, though. The Optimizing Model is an instance of such general strategy, inspired by Lee and Cummins' (2004) evidence accumulation model (based on Ratcliff's diffusion model). In Lee and Cummins' model the information gathered from the environment support each of alternatives with the strength that depends on some features of the cue considered. Optimizing model is based on the same idea but, in contrast, it is general, continuous, mathematical and has no free parameters.

The dispute between Simon and Anderson, relating to rationality of human behaviour, can be hardly solved solely in an empirical way, but some research may be support one of the stands in the dispute.

## The Model

Optimizing model assumes that the main goal of decision maker is reward maximization. The reward is equal to a difference between received payment and incurred costs. Therefore decision maker must satisfy two opposing subgoals: maximizing payment and minimising costs. As both subgoals inversely depend on the amount of information perceived from environment, these amounts must be optimized with regard to overall reward.

The function of cost subject to the amount of perceived information ($k(n)$) is, in the simplest case, linear and takes the form expressed by the formula $k(n) = cn$. The value of payment in the most of decision environments, that are considered in literature, is equal to one in case of success and equal to zero in case of failure, thus expected value of payment comes to the value of success' probability ($P(n)$), which takes a form of logarithmic dependence on the amount of perceived information and is expressed by the formula (1):

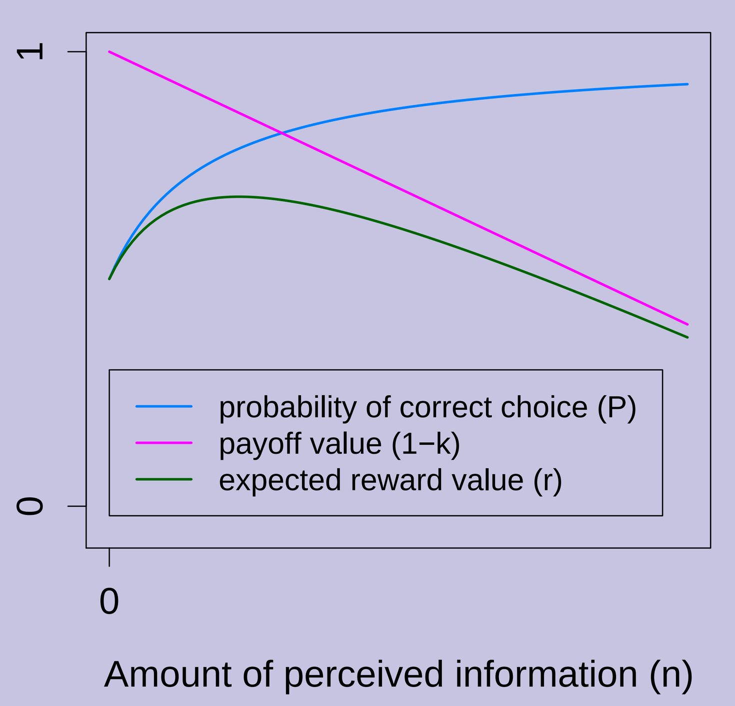
$$P(n) = 1 - \frac{\left(\frac{1}{2} - U(n)\right)^n}{2}, \tag{1}$$

where $U(n)$ is function, that returns probability of choosing proper alternative on the base of cue $n$ (e.g., "success"). After taking into account these two subgoals and possible decision cost ($K$), the expected overall reward ($r(n)$) can be computed on the basis of formula (2):

$$r(n) = P(n)\big(1 - k(n)\big) - K. \tag{2}$$

The dependence of these three functions on the amount of perceived information is shown in the figure .

## Expected reward



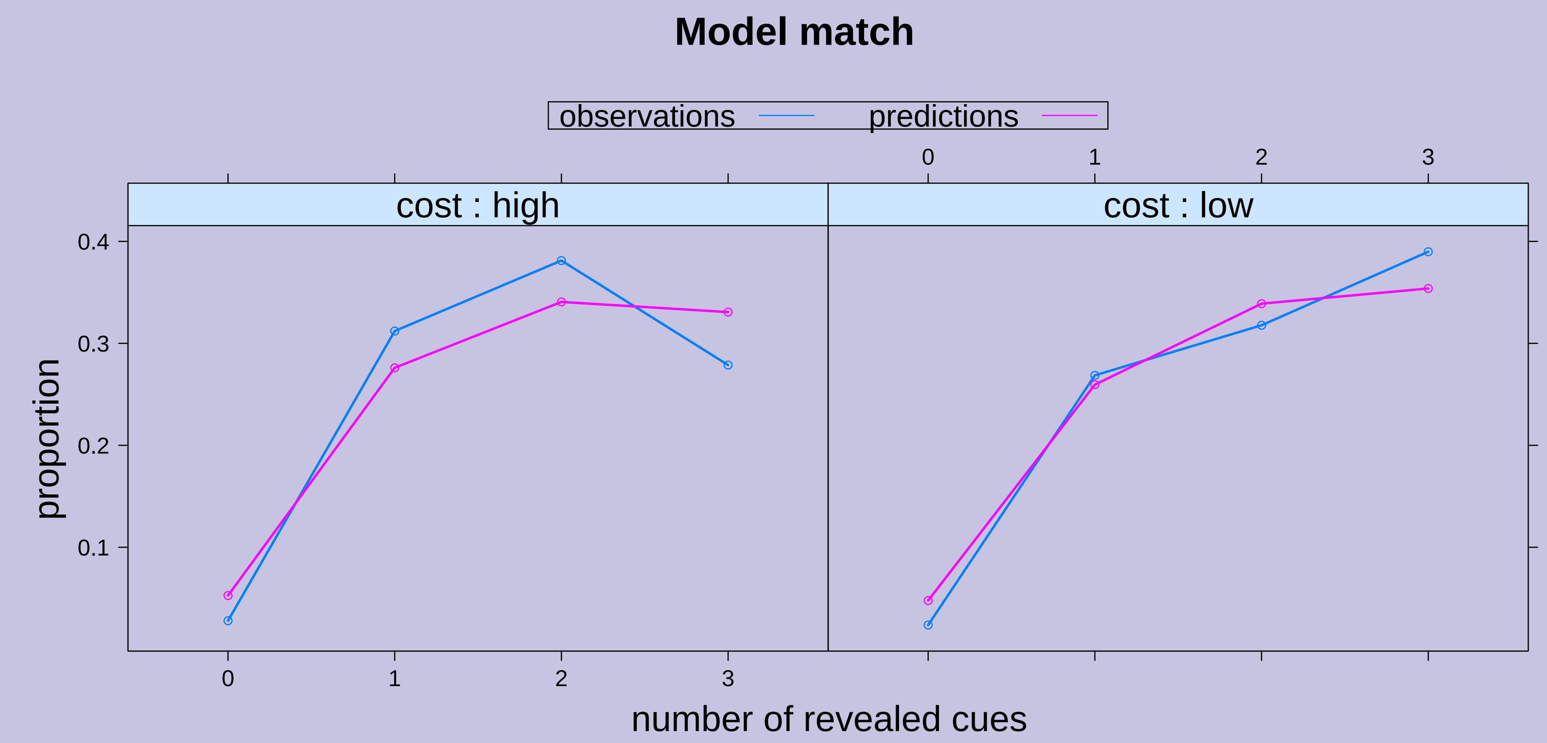

As people do not always choose the most promising altnernative, the probability matching rule must be regarded. Thus, the probability that agent will stop retrieving of information after $n$-th cue, is specified by formula (3), where $k$ is the number of cues and $\varepsilon$ is an error:

$$R(n) = \frac{\int_{n-0.5}^{n+0,5} r(n)dn}{\int_{0}^{k+0.5} r(n)dn} + \varepsilon. \tag{3}$$

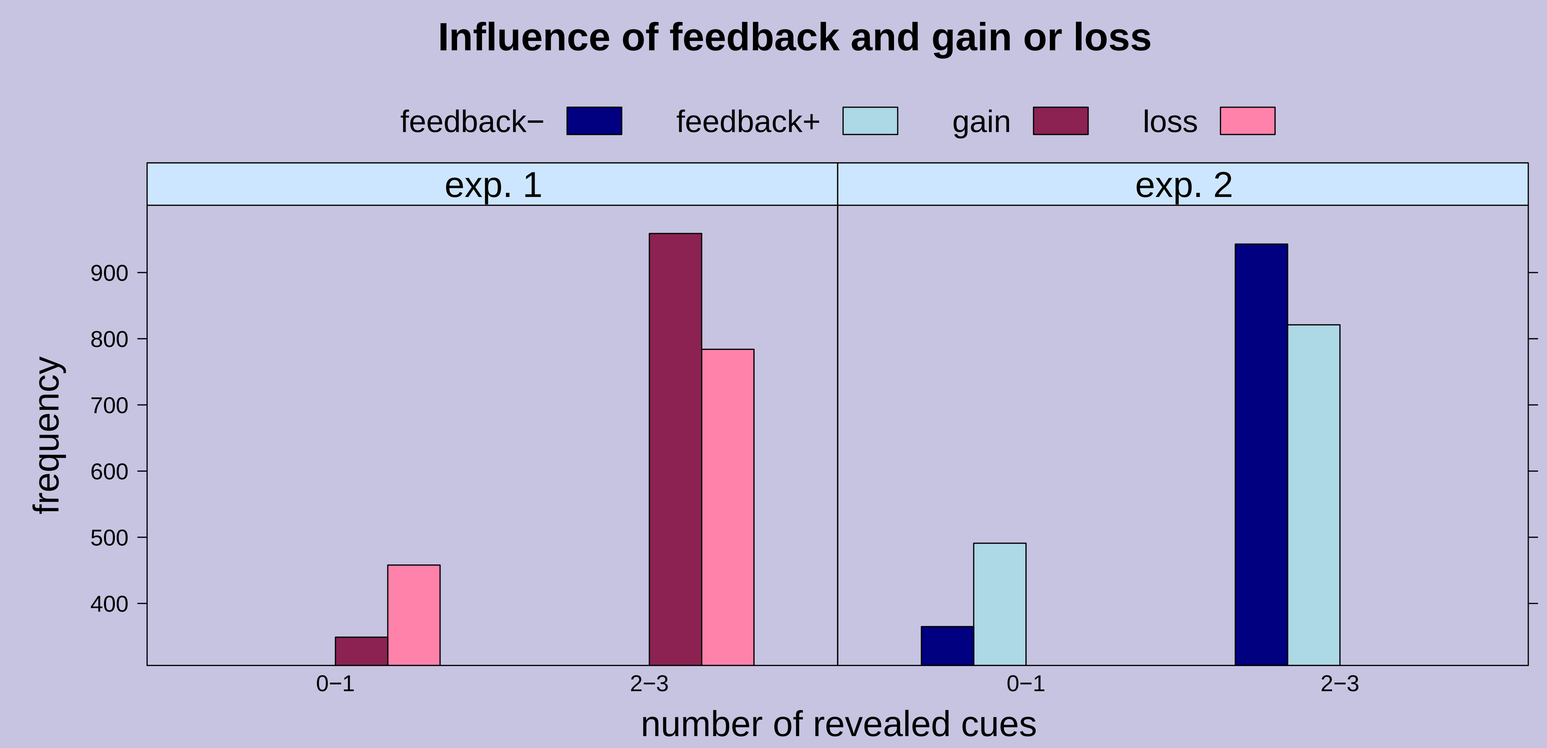
## Research

Three hypothesis were tested in two experiments, with the total of 48 subjects examined. (a) The general features of subjects' decision process would be consistent with what was predicted by the model. (b) Exp. I: In the situation of "gain" subjects would rather maximize the probability of correct decision, whereas in situation of "loss" they would rather minimize costs. (c) Exp. II: In lack of feedback condition, subjects would rather maximize the probability of correct decision, while in a more controllable condition, they would rather minimize costs. Two latter hypotheses were derived from the model with regard to the Kahneman and Tversky's prospect theory.

## Results




All three hypotheses have been confirmed. The match between the model predictions and subjects behaviour is good, especially taking into account the fact that the model has no free parameters (RMSD$= 0.12, r^2 = 0.95$). Also subjects have revealed more cues in conditions of gain in first experiment and lack of feedback in second experiment (both differences are significant: $p < 0.001$).




## Conclusions

The results provide a direct evidence for the hypothesis that the Optimizing Model is the accurate description of human decision process, and an indirect one, that it is possible to create a general and flexible strategy of decision making, which operates like one of the adaptive toolbox strategies, depending on how much information is available to it. What is more, the amount of information that is gained from the environment is not of arbitrary value, but results from optimization. Coherency between the optimization results and people behaviour also provides an indirect evidence that people act rationally in the sense pertaining to the environmental constraints

## References

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